

Chapter 3

The vertical structure of the atmosphere

In this chapter we discuss the observed vertical distribution of temperature, water vapor and greenhouse gases in the atmosphere. The observed temperature distribution is compared to the radiative equilibrium profile discussed in Chapter 2. We go on to calculate the implied distribution of pressure and density assuming the atmosphere to be in hydrostatic balance and compare with observations. We discover that the atmosphere does not have a distinct top. Rather, the density and pressure decay with height by a factor of e every 7 – 8 km.

3.1 Vertical distribution of temperature and ‘Greenhouse gases’

3.1.1 Typical temperature profile

Temperature varies greatly both vertically and horizontally throughout the atmosphere (as well as temporally). However, despite horizontal variations, the vertical structure of temperature is qualitatively similar everywhere, and so it is meaningful to think of (and to attempt to explain) a “typical” temperature profile. (We look at horizontal variations in Chapter 5.) A typical temperature profile (characteristic of 40°N in December) up to about 100 km is shown in Fig.3.1.

The profile is not governed by a simple law and is rather complicated.

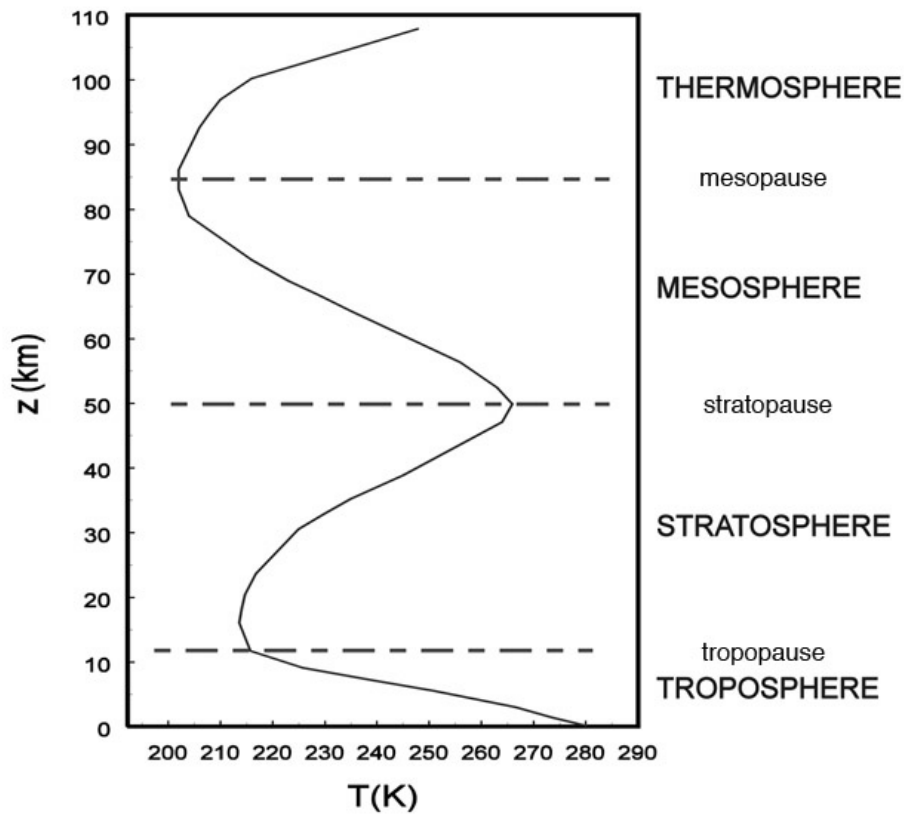


Figure 3.1: Vertical temperature profile for the 'US standard atmosphere' at $40^{\circ}N$ in December.

3.1. VERTICAL DISTRIBUTION OF TEMPERATURE AND ‘GREENHOUSE GASES’61

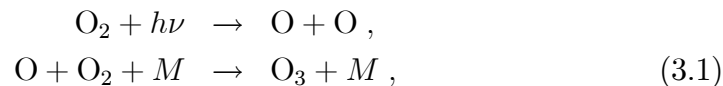
Note, however, that the (mass-weighted) mean temperature is close to 255 K, the emission temperature computed in the last chapter (remember almost all the mass of the atmosphere is in the bottom 10 km). The heating effect of solar radiation can be readily seen: there are 3 ‘hot spots’ corresponding to regions where solar radiation is absorbed at different wavelengths in the thermosphere, the stratopause and the troposphere. These maxima separate the atmosphere neatly into different layers.

3.1.2 Atmospheric layers

Coming down from the top of the atmosphere, the first ‘hot spot’ evident in Fig.3.1 is the **thermosphere**, where the temperature is very high and very variable. It is here that very short wavelength UV is absorbed by oxygen (*cf.*, Section 2.2) thus heating the region. Molecules (including O₂ as well as CO₂, the dominant IR emitter at this altitude) are dissociated (photolyzed) by high-energy UV ($\lambda < 0.1 \mu\text{m}$). Therefore, because of the scarcity of polyatomic molecules, IR loss of energy is weak, so the temperature of the region gets very high (as much as 1000 K). The air is so tenuous that assumptions of local thermodynamic equilibrium, as in black body radiation, are not applicable. At and above these altitudes, the atmosphere becomes ionized (the **ionosphere**), causing reflection of radio waves, a property of the upper atmosphere which is of great practical importance.

Below the *mesopause* at about 80 – 90 km altitude, temperature increases moving down through the **mesosphere** to reach a maximum at the *stratopause*, near 50 km, the second ‘hot spot’. This maximum is a direct result of absorption of medium wavelength UV (0.1 μm to 0.35 μm) by ozone. It is interesting to note that ozone concentration peaks much lower down in the atmosphere, at heights of 20 – 30 km, as illustrated in Fig.3.2. This is because the ‘‘ozone layer’’ is very opaque to UV (*cf* Fig.2.6) and so most of the UV flux is absorbed in the upper parts of the layer and there is little left to be absorbed at lower altitudes.

The reason for the existence of ozone at these levels is that it is produced here, as a by-product of the *photo-dissociation* (*photolysis*) of molecular oxygen, producing atomic oxygen which may then combine with molecular oxygen thus:



where $h\nu$ is the energy of incoming photons (ν is their frequency and h is Planck's constant) and M is any third body, needed to carry the excess energy.

The resulting ozone, through its radiative properties, is the reason for the existence of the **stratosphere**.¹ It is also one of the primary practical reasons to be interested in stratospheric behavior, since (as we saw in Chapter 2) ozone is the primary absorber of solar UV and thus shields life at the surface (including us) from the damaging effects of this radiation. The stratosphere, as its name suggests, is highly stratified, poorly mixed (*stratus* meaning layered) with long residence times for particles ejected into it (for example by volcanos) from the troposphere below. It is close to radiative equilibrium.

Below the *tropopause*, which is located at altitudes of 8–16 km (depending on latitude and season), temperature increases strongly moving down through the **troposphere** (*tropos* meaning ‘turn’) to the surface, the third ‘hot spot’. It contains about 85% of the atmosphere's mass and essentially all the water vapor, the primary greenhouse gas as illustrated in Fig.3.3. Note that the distribution of water vapor is, in large part, a consequence of the ‘Clausius-Clapeyron’ relation, Eq.(1.4), and rapidly decays with height as T decreases.

From the vertical distribution of O_3 and H_2O shown in Figs.3.2 and 3.3 and of CO_2 (which is well mixed in the vertical) a radiative equilibrium profile can be calculated, using the methods outlined in Chapter 2. This profile was shown in Fig.2.11. The troposphere is warmed in part through absorption of radiation by H_2O and CO_2 , the stratosphere is warmed, indeed created, through absorption of radiation by O_3 .

It is within the troposphere that almost everything we classify as “weather”



¹ Léon Philippe Teisserenc de Bort (1855-1913). French meteorologist who pioneered the use of unmanned high-flying instrumented balloons and discovered the stratosphere. He was the first to identify the temperature inversion at the tropopause. In 1902 he suggested that the atmosphere was divided into two layers.

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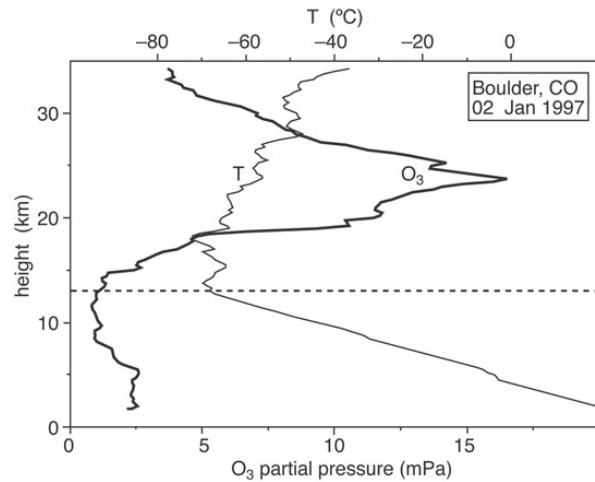


Figure 3.2: A typical winter ozone profile in middle latitudes (Boulder, CO, USA, 2 Jan 1997). The heavy curve shows the profile of ozone partial pressure (mPa), the light curve temperature ($^{\circ}\text{C}$) plotted against altitude up to about 33 km. The dashed horizontal line shows the approximate position of the tropopause. [Balloon data, courtesy of NOAA Climate Monitoring and Diagnostics Laboratory.]

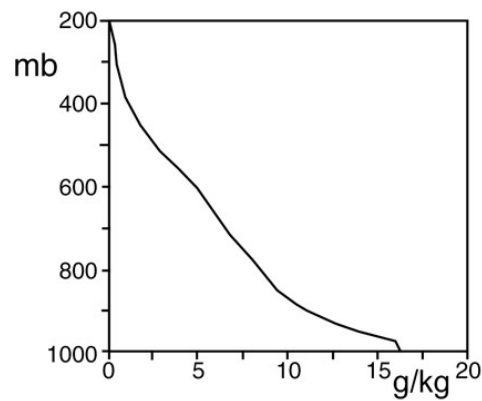


Figure 3.3: The global average vertical distribution of water vapor — in g kg^{-1} . — plotted against pressure.

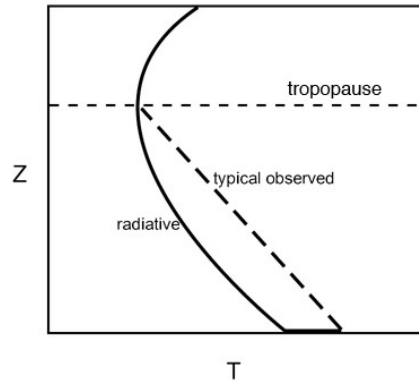


Figure 3.4: A schematized radiative equilibrium profile in the troposphere (cf Fig.2.11) (solid) and a schematized observed profile (dashed). Below the tropopause the troposphere is stirred by convection and weather systems and is not in radiative balance. Above the tropopause dynamical heat transport is of much lesser importance and the observed T is close to the radiative profile.

is located (and, of course, it is where we happen to live); it will be the primary focus of our attention. As we shall see in Chapter 4, its thermal structure cannot be satisfactorily explained solely on the basis of radiative balances. The troposphere is, in large part, warmed by convection carrying heat up from the lower surface. Temperature profiles observed in the troposphere, and as calculated from radiative equilibrium, are illustrated schematically in Fig.3.4. The observed profile is rather different from the radiative equilibrium profile of Fig.2.11. As noted at the end of Chapter 2, the large temperature discontinuity at the surface in the radiative equilibrium profile is not observed in practice. This discontinuity in temperature triggers a convective mode of vertical heat transport, which is the subject of Chapter 4.

Having described the observed T profile, we now go on to discuss the associated p and ρ profiles.

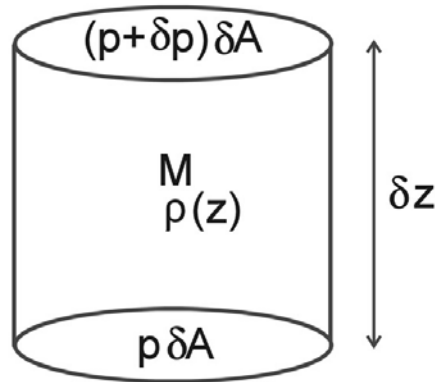


Figure 3.5: A vertical column of air of density ρ , horizontal cross-sectional area δA , height δz and mass $M = \rho \delta A \delta z$. The pressure on the lower surface is p , the pressure on the upper surface is $p + \delta p$.

3.2 The relationship between pressure and density: hydrostatic balance

Let us imagine that the atmospheric T profile is as observed, for example, in Fig.(3.1). What is the implied vertical distribution of pressure p and density ρ ? If the atmosphere were at rest – static – then pressure at any level would depend on the weight of the fluid above that level. This balance, which we now discuss in detail, is called *hydrostatic balance*.

Consider Fig.3.5 which depicts a vertical column of air of horizontal cross-sectional area δA and height δz .

Pressure $p(z)$ and density $\rho(z)$ of the air are both expected to be functions of height z (they may be functions of x, y , and t also). If the pressure at the bottom of the cylinder is $p_B = p(z)$, then that at the top is

$$\begin{aligned} p_T &= p(z + \delta z) \\ &= p(z) + \delta p, \end{aligned}$$

where δp is the change in pressure moving from z to $z + \delta z$. Assuming δz to be small,

$$\delta p = \frac{\partial p}{\partial z} \delta z. \quad (3.2)$$

Now, the mass of the cylinder is

$$M = \rho \delta A \delta z.$$

If the cylinder of air is not accelerating, it must be subjected to zero net force. The vertical forces (upward being positive) are:

- i) gravitational force $F_g = -gM = -g\rho\delta A\delta z$,
- ii) pressure force acting at the top face, $F_T = -(p + \delta p)\delta A$, and
- iii) pressure force acting at the bottom face, $F_B = p\delta A$.

Setting the net force $F_g + F_T + F_B$ to zero gives $\delta p + g\rho\delta z = 0$, and using Eq.(3.2) we obtain:

$$\frac{\partial p}{\partial z} + g\rho = 0 \quad (3.3)$$

Eq.(3.3) is the equation of **hydrostatic balance**. It describes how pressure decreases with height in proportion to the weight of the overlying atmosphere. Note that since p must vanish as $z \rightarrow \infty$ — the atmosphere fades away² — we can integrate Eq.(3.3) from z to ∞ to give the pressure at any height

$$p(z) = g \int_z^\infty \rho dz. \quad (3.4)$$

Here $\int_z^\infty \rho dz$ is just the mass per unit area of the atmospheric column above z . The surface pressure is then related to the total mass of the atmosphere



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Blaise Pascal (1623-1662), a physicist and mathematician of prodigious talents and accomplishments, was also intensely interested in the variation of atmospheric pressure with height and its application to the measurement of mountain heights. In 1648 he observed that the pressure of the atmosphere decreases with height and, to his own satisfaction, deduced that a vacuum existed above the atmosphere. The unit of pressure is named after him.

above: Eq.(3.4) implies that $p_s = \frac{gM_a}{\text{surface area of Earth}} = 1013\text{h Pa}$, using the data in Tables 1.2 and 1.3. This indeed is the global average surface pressure.

The only important assumption made in the derivation of Eq.(3.3) was the neglect of any vertical acceleration of the cylinder (in which case, the net force need not be zero). This is an excellent approximation under almost all circumstances in both the atmosphere and ocean. It can become suspect, however, in very vigorous small-scale systems in both the atmosphere and ocean (*e.g.*, convection, tornados, violent thunderstorms and deep, polar convection in the ocean — see Chapters 4 and 11). We discuss hydrostatic balance in the context of the equations of motion of a fluid in Section 6.2.3.

Note that Eq.(3.3) does not tell us what $p(z)$ is, since we do not know *a priori* what $\rho(z)$ is. In order to determine $p(z)$ we must invoke an equation of state to tell us the connection between ρ and p , as described in the next section.

3.3 Vertical structure of pressure and density

Using the equation of state of air, Eq.(1.1), we may rewrite Eq.(3.3) as

$$\frac{\partial p}{\partial z} = -\frac{gp}{RT}. \quad (3.5)$$

In general, this has not helped, since we have replaced the 2 unknowns p and ρ by p and T . However, unlike p and ρ , which vary by many orders of magnitude from the surface to, say, 100 km altitude, the variation of T is much less. In the profile in Fig.3.1, for example, T lies in the range 200 – 280 K, thus varying by more than 15% from a value of 240 K. So, for the present purpose, we may replace T by a typical mean value to get a feel for how p and ρ vary.

3.3.1 Isothermal atmosphere

If $T = T_0$, a constant, we have

$$\frac{\partial p}{\partial z} = -\frac{gp}{RT_0} = -\frac{p}{H},$$

where H , the *scale height*, is a constant (neglecting, as noted in Chapter 1, the small dependence of g on z) with the value

$$H = \frac{RT_0}{g}. \quad (3.6)$$

If H is constant, the solution for p is, noting that by definition, $p = p_s$ at the surface $z = 0$,

$$p(z) = p_s \exp\left(-\frac{z}{H}\right). \quad (3.7)$$

Alternatively, by taking the logarithm of both sides we may write z in terms of p thus:

$$z = H \ln\left(\frac{p_s}{p}\right). \quad (3.8)$$

Thus pressure decreases upward exponentially with height, with e -folding height H . For the troposphere, if we choose a representative value $T_0 = 250$ K, then $H = 7.31$ km. Therefore, for example, in such an atmosphere p is 100h Pa, i.e. one-tenth of surface pressure, at a height of $z = H \times (\ln 10) = 16.83$ km. This is quite close to the observed height of the 100h Pa surface. Note, very roughly, the 300h Pa surface is at a height of about 9 km and the 500h Pa surface at a height of about 5.5 km.

3.3.2 Non-isothermal atmosphere

What happens if T is not constant? In this case we can still define a local scale height

$$H(z) = \frac{RT(z)}{g}, \quad (3.9)$$

such that

$$\frac{\partial p}{\partial z} = -\frac{p}{H(z)}$$

where $H(z)$ is the local scale height. Therefore

$$\frac{1}{p} \frac{\partial p}{\partial z} = \frac{\partial \ln p}{\partial z} = -\frac{1}{H(z)}. \quad (3.10)$$

whence

$$\ln p = \int_0^z \frac{dz'}{H(z')} + \text{constant},$$

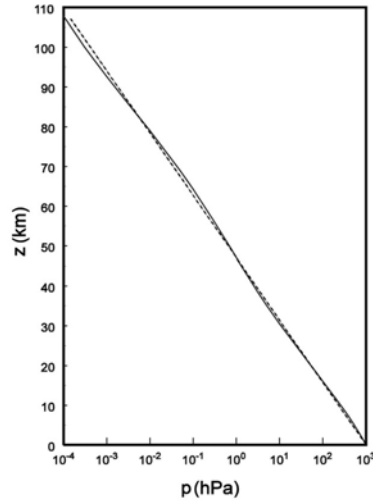


Figure 3.6: Observed profile of pressure (solid) plotted against a theoretical profile (dashed) based on Eq.(3.7) with $H = 6.8$ km.

or

$$p(z) = p_s \exp \left(- \int_0^z \frac{dz'}{H(z')} \right). \quad (3.11)$$

Note that if $H(z) = H$, the constant value considered in the previous section, Eq.(3.11) reduces to Eq.(3.7).

In fact, despite its simplicity, the isothermal result, Eq.(3.7), yields profiles that are a good approximation to reality. Fig.3.6 shows the actual pressure profile for 40°N in December (corresponding to the temperature profile in Fig.3.1) [solid], together with the profile given by Eq.(3.7), with $H = 6.80$ km [dashed]. Agreement between the two is generally good (to some extent, the value of H was chosen to optimize this). The differences can easily be understood from Eqs.(3.9) and (3.10). In regions where, for example, temperatures are warmer than the reference value ($T_0 = gH/R = 237.08$ K for $H = 6.80$ km) — such as (cf. Fig.3.1) in the lower troposphere, near the stratopause and in the thermosphere — the observed pressure decreases less rapidly with height than predicted by the isothermal profile.

3.3.3 Density

For the isothermal case, the density profile follows trivially from Eq.(3.7), by combining it with the gas law Eq.(1.1):

$$\rho(z) = \frac{p_s}{RT_0} \exp\left(-\frac{z}{H}\right). \quad (3.12)$$

Thus, in this case, density also falls off exponentially at the same rate as p . One consequence of Eq.(3.12) is that, as noted at the start of Chapter 1, about 80% of the mass of the atmosphere lies below 10 km.

For the non-isothermal atmosphere with temperature $T(z)$, it follows from Eq.(3.11) and the equation of state that

$$\rho(z) = \frac{p_s}{RT(z)} \exp\left(-\int_0^z \frac{dz'}{H(z')}\right). \quad (3.13)$$

3.4 Further reading

A thorough discussion of the role of the various trace gases in the atmospheric radiative balance can be found in Chapter 3 of Andrews (2000).

3.5 Problems

1. Use the hydrostatic equation to show that the mass of a vertical column of air of unit cross-section, extending from the ground to great height, is $\frac{p_s}{g}$, where p_s is the surface pressure. Insert numbers to estimate the mass on a column of air of area 1 m^2 . Use your answer to estimate the total mass of the atmosphere.
2. Using the hydrostatic equation, derive an expression for the pressure at the center of a planet in terms of its surface gravity, radius a and density ρ , assuming that the latter does not vary with depth. Insert values appropriate for the earth and evaluate the central pressure. [Hint: the gravity at radius r is $g(r) = \frac{Gm(r)}{r^2}$ where $m(r)$ is the mass inside a radius r and $G = 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$ is the gravitational constant. You may assume the density of rock is 2000 kg m^{-3} .]

3. Consider a horizontally uniform atmosphere in hydrostatic balance. The atmosphere is isothermal, with temperature of -10°C . Surface pressure is 1000 mbar.
 - (a) Consider the level that divides the atmosphere into two equal parts by mass (*i.e.*, one-half of the atmospheric mass is above this level). What is the altitude, pressure and density at this level?
 - (b) Repeat the calculation of part (a) for the level below which lies 90% of the atmospheric mass.

4. Derive an expression for the hydrostatic atmospheric pressure at height z above the surface in terms of the surface pressure p_s and the surface temperature T_s for an atmosphere with constant lapse rate of temperature $\Gamma = -\frac{dT}{dz}$. Express your results in terms of the dry adiabatic lapse rate $\Gamma_d = \frac{g}{c_p}$ — see Section 4.3.1. Calculate the height at which the pressure is 0.1 of its surface value (assume a surface temperature of 290 K and a uniform lapse rate of 10 K km^{-1}).

5. Spectroscopic measurements show that a mass of water vapor of more than 3 kg m^{-2} in a column of atmosphere is opaque to the ‘terrestrial’ waveband. Given that water vapor typically has a density of 10^{-2} kg m^{-3} at sea level (see Fig.3.3) and decays in the vertical like $e^{-\left(\frac{z}{b}\right)}$, where z is the height above the surface and $b \sim 3\text{ km}$, estimate at what height the atmosphere becomes transparent to terrestrial radiation.

By inspection of the observed vertical temperature profile shown in Fig.3.1, deduce the temperature of the atmosphere at this height. How does it compare to the emission temperature of the Earth, $T_e = 255\text{ K}$, discussed in Chapter 2? Comment on your answer.

6. Make use of your answer to Q.1 of Chapter 1 to estimate the error incurred in p at 100 km through use of Eq.(3.11) if a constant value of g is assumed.

