# Circulation of the Atmosphere and Ocean: an introductory text

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## 0.1 Outline, scope and rationale of the book

This is an introductory text on the circulation of the atmosphere and ocean with an emphasis on global scales. It has been written for undergraduate students who have no prior knowledge of meteorology and oceanography or training in fluid mechanics. We believe that the text will also be of use to beginning graduate students in the field of atmospheric, oceanic and climate science. By the end of the book we hope that readers will have a good grasp of what the atmosphere and ocean look like on the large-scale, and, through application of the laws of mechanics and thermodynamics, why they look like they do. We will also be able to place our observations and understanding of the present climate in the context of how climate has evolved and changed over earth history.

The book is roughly divided in to three equal parts. The first third deals exclusively with the atmosphere (Chapters 1 to 5), the last third with the ocean and its role in climate (Chapters 9 to 12). Sandwiched in between we develop the necessary fluid dynamical background (Chapter 6 and 7). Our discussion of the general circulation of the atmosphere (Chapter 8), follows the dynamical chapters. The text can be used in a number of different ways. It has been written so that those interested primarily in the atmosphere might focus on Chapters 1 to 8. Those interested in the ocean can begin at Chapter 9, referring back as necessary to the dynamical Chapters 6 and 7. It is our hope, however, that many will be interested in learning about both fluids. Indeed, one of the joys of working on this text — and using it as background material for undergraduate courses taught at the Massachusetts Institute of Technology (MIT) — has been our attempt to discuss the circulation of the atmosphere and ocean in a single framework and in the same spirit.

In our writing we have been led by the observations, rather than theory. We have not written a book about fluid dynamics illustrated by atmospheric and oceanic phenomena. Rather we hope that the observations take the lead and theory is introduced as and when it is needed. Advanced dynamical ideas are only made use of if we deem it essential to bring order to the observations. We have also chosen not to unnecessarily formalize our discussion. Yet, as far as is possible, we have offered rigorous physical discussions expressed in mathematical form: we build (nearly) everything up from first principles, our explanations of the observations are guided by theory, and these guiding principles are, we hope, clearly espoused.

The vast majority of the observations described and interpreted here are

#### 0.2. PREFACE

available electronically via the associated website. We make much use of the remarkable data-base and web-browsing facilities developed at the Lamont Doherty Earth Observatory of Columbia University. Thus the raw data presented by figures on the pages of the book can be accessed and manipulated over the web, as described in an Appendix.

One particularly enjoyable aspect of the courses from which this book sprang have been the numerous laboratory experiments carried out in lectures as demonstrations, or studied in more detail in undergraduate laboratory courses. We hope that some of this flavor comes through on the written page. We have attempted to weave the experiments into the body of the text so that, in the spirit of the best musicals, the 'song and dance routines' seem natural rather than forced. The experiments we have chosen to describe are simple and informative and, for the most part, do not require sophisticated apparatus. Video loops of the experiments can be viewed over the web, but there is no real substitute for carrying them out oneself. We encourage you to try. Details of the equipment required to carry out the experiments, including the necessary rotating turntables, can be found in Section 13.4.

Before getting on to the meat of our account, we now make some introductory remarks about the nature of the problems we are concerned with.

# 0.2 Preface

The circulation of the atmosphere and oceans is inherently complicated, involving the transfer of radiation through a semi-transparent medium of variable composition, phase changes between liquid water, ice and vapor, interactions between phenomena on scales from centimeters to the globe, and time-scales from seconds to millennia. But one only has to look at a picture of the Earth from space, such as that shown in Fig.1, to appreciate that organizing principles must be at work to bring such order and beauty.

This book is about the large-scale circulation of the atmosphere and ocean and the organizing fluid-mechanical principles that shape it. We will learn how the unusual properties of rotating fluids manifest themselves in, and profoundly influence, the circulation of the atmosphere and ocean and the climate of the planet. The necessary fluid dynamics will be developed and explored in the context of phenomena that play an important role in climate, such as convection, weather systems, the Gulf Stream and the thermohaline circulation of the ocean. Extensive use is made of laboratory experiments designed to isolate and illustrate key ideas. Any study of climate dynamics would be incomplete without a discussion of radiative transfer theory and so we will also cover fundamental ideas on energy balance. In the final chapters we also discuss the interaction of the atmosphere and ocean and how the two collude together to control the climate of the earth. The paleoclimate record suggests that the climate of the past has been very different from that of today. Thus we use the understanding gleaned from our study of the present climate to speculate on mechanisms that might drive climate change.

In these introductory remarks we draw out those distinctive features of the fluid mechanics of the atmosphere and ocean that endow its study with a unique flavor. We are dealing with what can be called 'natural' fluids which are energized thermally on a differentially-heated sphere whose rotation tightly constrains the motion.

#### 0.2.1 Natural fluid dynamics

Fluid dynamics is commonly studied in Engineering and Applied Mathematics Departments. A typical context might be the following. In a fluid of constant density, shearing eddies develop whenever circumstances are such as to force a strong shear (velocity contrast over a short distance). For example, flow past a solid obstacle leads to a turbulent wake (see Fig.2) as in the flow of water down a stream or in air blowing over an airfoil. The kinetic energy of the eddying motion comes from the kinetic energy of steady flow impinging on the obstacle.

One can study the problem experimentally (by constructing a laboratory analog of the motion) or by solving (rather complicated) differential equations. However, shear-induced turbulence, and indeed much of classical hydrodynamics<sup>1</sup>, is not *directly* applicable to the fluid dynamics of the atmosphere and ocean because it assumes that the density,  $\rho$ , is constant, or more precisely, that the density only depends on the pressure<sup>2</sup>, p; i.e.  $\rho = \rho(p)$ . The energy source for the "eddies" that form the turbulent wake in Fig.2 comes from the kinetic energy of the incoming steady stream. There

<sup>&</sup>lt;sup>1</sup>See, for example, the treatise on classical hydrodynamics by Horace Lamb: Hydrodynamics, Cambridge, 1932.

<sup>&</sup>lt;sup>2</sup>A fluid in which the density depends only on the pressure,  $\rho = \rho(p)$ , is called a barotropic fluid. A fluid in which the density depends on both temperature and pressure,  $\rho = \rho(p, T)$ , is called a boroclinic fluid.



Figure 1: A view of earth from space over the north Pole. The Arctic ice cap can be seen in the center. The white swirls are clouds associated with atmospheric weather patterns. Courtesy of NASA/JPL.



Figure 2: Schematic diagram showing a fluid of constant density flowing past a solid obstacle, as might happen in the flow of water down a stream. Shearing eddies develop in a thin layer around the obstacle and result in a turbulent wake in the lee of the obstacle. The kinetic energy of the eddying motion comes from the kinetic energy of the steady flow impinging on the obstacle.



Figure 3: A mosaic of satellite images showing the water vapor distribution over the globe at a height between 6 - 10 km above the surface. We see the organization of H<sub>2</sub>O by the circulation; dry (sinking) areas in the subtropics (around  $\pm 30^{\circ}$ ) are dark, moist (upwelling) regions of the equatorial band are bright. Jet streams of the middle latitudes appear as elongated dark regions with adjacent clouds and bright regions. From NASA.



Figure 4: Due to the overwhelming importance of gravity, pressure increases downward in the atmosphere and ocean. For gravitational stability, density must also increase downward — as sketched in the diagram — with heavy fluid below and light fluid above. If  $\rho = \rho(p)$  only, then the density is independent of T and so the fluid cannot be brought in to motion by heating and/or cooling.

is a superficial resemblance to the ubiquitous large-scale eddies and swirls observed in the atmosphere, beautifully revealed by the water vapor images shown in Fig.3. However the energy for the eddies seen in Fig.3 come directly, or indirectly, from thermal, rather than mechanical sources.

Let us consider for a moment what the atmosphere or ocean would be like if it were made up of a fluid in which the density is independent of temperature and so can be written  $\rho = \rho(p)$ . Because of the overwhelming influence of gravity, pressure increases downward in the atmosphere and ocean. If the arrangement is to be stable, light fluid must be on top of heavy fluid, and so the density must also increase downward — as sketched in Fig.4. Now if  $\rho$  does not depend on T, the fluid cannot be brought in to motion by heating/cooling. We conclude that if we make the assumption  $\rho = \rho(p)$  then *life is abstracted out of the fluid* because it cannot convert thermal energy into kinetic energy. But everywhere around us in the atmosphere and ocean we find fluid doing just that: acting as a natural heat engine, generating and maintaining its own motion by converting thermal energy in to kinetic energy. The fluid can only do this because  $\rho$  is not just a function of p. If  $\rho$  depends on both pressure and temperature<sup>3</sup>,  $\rho = \rho(p, T)$ , as sketched in

<sup>&</sup>lt;sup>3</sup>The density of the ocean depends on salinity, as well as temperature and pressure:  $\rho = \rho(p, S, T)$ . Then, for example, convection can be triggered from the surface layers of the ocean by the formation of ice; fresh water is locked up in the ice, leaving brackish and



Figure 5: In contrast to Fig.2, if  $\rho$  depends on both pressure and temperature,  $\rho = \rho(p, T)$ , then fluid heated by the sun, for example, can become buoyant and rise in convection. Such a fluid can be energized thermally.

Fig.5, then fluid heated by the sun, for example, can become buoyant and rise in convection. Such a fluid can convect by converting thermal energy in to kinetic energy — it is *full of life* because it can be energized thermally.

In meteorology and oceanography we are always dealing with fluids in which  $\rho = \rho(p, T, ...)$  and turbulence is, in the main, thermally rather than mechanically maintained. Rather than classical hydrodynamics we are concerned with 'natural aerodynamics' or 'geophysical fluid dynamics', the fluid dynamics of real fluids. The latter phrase, often going by the shorthand 'GFD', is now widely used to describe the kind of fluid dynamics we are dealing with.

#### 0.2.2 Rotating fluid dynamics: GFD Lab 0

In climate dynamics we are not only dealing with the natural fluids described above, but also, because Earth is a rather rapidly rotating planet, with rotating fluids. As we shall see during the course of our study, rotation endows fluids with remarkable properties.

If one looks up the definition of 'fluid' in the dictionary one finds:

something that can "change rapidly or easily" or "fill any container into which it is poured."

hence heavy water behind at the surface.

But fluid on a rotating planet cannot move in arbitrary paths because, if the scales of motion are sufficiently large and sluggish, they are profoundly aware of and affected by rotation. In a very real sense the atmosphere and ocean are not "fluid" at all; they are tightly constrained by the rotation of the Earth. This constraint makes the two fluids much more similar than one might expect — the atmosphere and ocean can, and we would argue should, be studied together. This is what we set out to do in this text.

The unusual properties of rotating fluids can be demonstrated in the following very simple laboratory experiment.

#### GFD Lab 0; Rigidity imparted to rotating fluids

We take two tanks and place one on a rotating table and the other on a desk in the laboratory. We fill them with water to a depth of  $\sim 20 \,\mathrm{cm}$  or so, set the rotating table turning at a speed of about 15-20 revolutions per minute (see Appendix for discussion of rotating table) and leave them to settle down for 15 minutes or so. We gently agitate the two bodies of water — one's hand is best, using an up-down beating motion (try not to introduce a systematic swirl) — to generate motion, and wait ( $\leq 1 \text{ minute}$ ) for things to settle down a little, but not so long that the currents to die away. We observe the motion by introducing dye (food coloring).

In the non-rotating tank the dye disperses much as we might intuitively expect. But in the rotating body of water something glorious happens. We see beautiful streaks of dye falling vertically; the vertical streaks become drawn out by horizontal fluid motion into vertical 'curtains' which wrap around one-another. Try two different colors and watch the interleaving of fluid columns — see Fig.6 here and Fig.7.7 of Chapter 7.

The vertical columns, which are known as 'Taylor columns' after G.I. Taylor who discovered them (see Chapter 7), are a result of the rigidity imparted to the fluid by the rotation of the tank. The water moves around in vertical columns which are aligned parallel to the rotation vector. It is in this sense that rotating fluids are rigid. As the horizontal spatial scales and timescales lengthen, rotation becomes an increasingly strong constraint on the motion of both the atmosphere and ocean.

On what scales might the atmosphere, ocean, or our laboratory experiment, 'feel' the effect of rotation? Suppose that typical horizontal currents (atmospheric or oceanic, measured, as they are, in the rotating frame) are given by U, and the typical length scale over which the current varies is



Figure 6: Taylor columns revealed by food coloring in the rotating tank. The water is allowed to come into solid body rotation and then gently stirred by hand. Dyes are used to visualize the flow. At the top we show the rotating cylinder of water with curtains of dye falling down from the surface. Below we show the beautiful patterns of dyes of different colors being stirred around one another by the rotationally constrained motion.

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L. Then the timescale of the motion is L/U. Let's compare this with  $\tau_{rot}$ , the period of rotation, by defining a non-dimensional number (known as the Rossby number — again, see Section 7.1):

$$R_o = \frac{U \times \tau_{rot}}{L}$$

If  $R_o$  is much greater than one, then the timescale of the motion is short relative to a rotation period and so rotation will not significantly influence the motion. If  $R_o$  is much less than one, then the motion will be aware of rotation.

In our laboratory tank we observe horizontal swirling motions of perhaps  $U \sim 1 \text{ cm s}^{-1}$  over the scale of the tank  $L \sim 30 \text{ cm}$ , which is rotating with a period  $\tau_{\text{tank}} = 3 \text{ s}$ . This yields a Rossby number for the tank flow of  $R_{o_{\text{tank}}} = 0.1$ . Thus rotation will be an important constraint on the fluid motion, as we have witnessed by the presence of Taylor columns in Fig.6.

Let us estimate  $R_o$  for large-scale flow in the atmosphere and ocean.

- ATMOSPHERE (for a weather system, say), discussed in Chapter 5 and 7:  $L \sim 5000 \,\mathrm{km}, U \sim 10 \,\mathrm{m \, s^{-1}}$ , and  $\tau_{rot} = 1 \,\mathrm{d} \approx 10^5 \,\mathrm{s}$ , giving  $R_{o_{atmos}} = 0.2$ , which suggests that rotation will be important.
- OCEAN (for the great gyres of the Atlantic or Pacific Oceans, for example, described in Chapter 9):  $L \sim 1000$  km,  $U \sim 0.1$  ms<sup>-1</sup>, giving  $R_{o_{ocean}} = 0.01$ , and rotation will be a controlling factor.

It is clear, then, that rotation will be of paramount importance in shaping the pattern of air and ocean currents on sufficiently large scales. Indeed much of the structure and organization seen in Fig.1 is shaped by rotation.

#### 0.2.3 Holicism

There is another aspect that gives our study of climate dynamics its distinctive flavor. The climate is a unity. Only if great care is taken can it be broken up and the parts studied separately, since every aspect affects every other aspect. To illustrate this point let us consider the interplay between the transfer of radiation through the atmosphere (known as radiative transfer) and the fluid motion. As we shall see in Chapter 2 and 3, the radiative temperature profile depends on, amongst other things, the distribution of water vapor because water vapor strongly absorbs radiation in those wavelengths



Figure 7: A schematic diagram showing the interplay between radiative transfer and circulation. The absorption of radiation by the atmosphere is very sensitive to the distribution of water vapor. But the water vapor distribution depends on the motion which in turn depends on the heating, completing a closed cycle.

at which the earth is a principal radiator of energy. But the water vapor distribution is not given because it depends on the motion, as can be clearly seen in Fig.1. The motion in turn depends on the heating, which depends on the radiative transfer. The closed cycle sketched in Fig.7 must be studied as a whole.

So, the background may be ordinary physics — classical mechanics and thermodynamics applied to a fluid on a rotating, differentially-heated sphere — but the study of the whole process has its own distinct and unique flavor. The approach is HOLISTIC rather than reductionist, because there is never a single cause. If one asks the question: "Why do swallows migrate south in the autumn?" String theory will never give us the answer; this kind of science may......